Paper / Subject Code: N2283111 / Signals and systems (Mathematics-III)

SE Sem 3 NEP Reg Winter 2025 - ECS 17.12.2025

(Time: 02 HOURS) (MAX. MARKS: 60)

Note:
1. Question **No. 1** is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

			Marks
Q.1		Answer the following **(Any five)**	
	a.	Given $f(t)=\frac{\sin t}{t}$, find the Laplace transform of $f'(t)$	03
	b.	Let V be the set of all ordered pairs of real numbers. Addition and multiplication are defined by $(x_1,y_1)+(x_2,y_2)=(x_1+x_2,y_1+y_2)$ and $\alpha(x,y)=(x,\alpha y)$ where $x,y,x_1,y_1,x_2,y_2,\alpha$ are real numbers. Show that V is not a vector space over $\mathbb{R}$, where $\mathbb{R}$ is a set of real numbers.	03
	c.	The half-range Fourier sine series for the function $f(x)$ is given by $f(x)=\sum_{n=1}^{\infty} b_n \sin nx$. Find the coefficient b_1 for the function $f(x)=x\sin x$ in $(0,\pi)$.	03
	d.	Find $L^{-1}\left[\frac{s+2}{s^2+4s+7}\right]$	03
	e.	If $A=\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ where a,b,c are positive integers, then prove that $a+b+c$ is an eigen value of A.	03
	f.	Determine whether the mapping $T:\mathbb{R}^2\to\mathbb{R}^3$ defined as $T(x,y)=(x,y,0)$ is linear or not over $\mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers.	03
	g.	Find the Laplace transform of $\int_0^t e^{-4u}\sin 3u\,du$	03
Q.2	a.	Prove that $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt=\frac{1}{4}\log 5$	05
	b.	Prove that, the vectors $(1,2,1)$, $(2,1,0)$ and $(1,-1,2)$ in $\mathbb{R}^3$ are linearly independent, where $\mathbb{R}$ is a set of real numbers.	05
	c.	Show that the transformation $T:\mathbb{R}^2\to\mathbb{R}^3$ defined as $T(a,b)=(a+b,a-b,\ b)$, for all $a,b\in\mathbb{R}$ is linear, where $\mathbb{R}$ is the set of real numbers. Find its kernel and nullity.	05

(80)

Q.3 a. Find the Fourier series representing $f(x) = x,\ 0 < x < 2\pi$ 05

b. Find the inverse Laplace transform of $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$ 05

c. If $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, find A^{50}. 05

Q.4 a. Find the Fourier expansion of $f(x) = \begin{cases} 0, & -2 < x < -1 \\ 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ 08

b. Evaluate the following integral by using Laplace transform 07
$\int_0^\infty e^{-2t} \left(\int_0^t \frac{e^{-u} \sin u}{u} du \right) dt$

Q.5 a. Is the matrix $A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$ diagonalisable? If so, find the diagonal form and the transforming matrix. 08

b. Let $V = \mathbb{R}^3$ and $W = \mathbb{R}^2$, where $\mathbb{R}$ is the set of real numbers. Let $T: V \to W$ be a linear transformation defined by $T(a,\ b,\ c) = (a,\ b)$. Find the matrix of T with respect to the bases $B_V = \{(1,-1,0), (1,0,\ -1), (1,1,1)\}$ and $B_W = \{(4,3), (3,2)\}$. 07

Q.6 a. Find the inverse Laplace transform of the following by convolution theorem: $\frac{(s+3)^2}{(s^2+6s+5)^2}$ 08

b. Find the orthonormal basis of $\mathbb{R}^3$ with standard inner product using Gram-Schmidt orthogonalization to the vectors $\alpha_1 = (1,0,1), \alpha_2 = (1,2,-2), \alpha_3 = (2,-1,1)$. 07

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